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Outlining a Non-Possible-Worlds-Based Conception of Modality [1]

There are different degrees of necessity. The proposition *metal expands when heated* is necessary in the sense that its truth is guaranteed by the laws of physics. Thus it is *nomically* necessary.

In every day speech the word "necessary" sometimes denotes a property weaker than nomic necessity. It is not uncommon to hear statements like: "if Bob lived in Albania for twenty years, then he necessarily speaks at least some Albanian." Here the word "necessarily" seems to mean "extremely probable".

In this paper I will be concerned with a kind of necessity that is stronger than even nomic necessity. Nomic necessity is necessity *relative* to the laws of nature that in fact hold. We feel that the natural laws that *do* hold might *not* hold. So nomic necessity is necessity relative to something that is not *itself* necessary. Thus there is a sense in which a proposition that is nomically necessary is not necessary at all.

The essay is concerned with the *strongest* kinds of necessity. 2+2 *must* equal 4. The necessity here is unconditional. It isn't that *relative* to certain facts about the world, 2+2 must equal four. There is nothing relative about this kind of necessity; no condition has to be satisfied; it is necessary all by itself, so to speak. It is thus necessary in the strongest possible sense.

In addition to being unconditionally necessary, *2+2=4* is also a priori. To know it, we merely examine concepts; we don't do empirical work. Kripke discovered that some unconditionally necessary propositions are *not* a priori.[2] Given that Hesperus *is* Phosphorous, it is impossible, in the strongest sense, that Hesperus should be anything *other* than Phosphorous. For it is unconditionally necessary that each thing is self-identical. But *Hesperus is Phosphorous* is a posteriori: astronomical work was needed to

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[1] I would like to thank an anonymous reviewer at *Metaphysica* for his incisive comments both on the historical precedents for my view and also on the logical structure of my argument.

[2] Kripke 1972.

learn that it is true. So here we have a proposition that is both unconditionally necessary and also a posteriori.

This paper is concerned exclusively with necessity in the strongest sense. We will not discuss the conditional necessity characteristic of propositions like *metals expand when heated*. So henceforth the words "necessity" and "necessarily" will, without exception, denote *only* necessity in the strongest sense.

Philosophers have long debated what it is for a proposition to be necessarily true. One approach is given by the following thesis:

(LC) A proposition is necessarily true iff it is logically (or conceptually) true.

So, for example, the proposition *triangles have three sides* is necessarily true because it is logically or conceptually true ("logico-conceptually" true). It is true wholly in virtue of the concepts composing it and of the way in which these concepts are arranged in that proposition. (Sometimes I will say "analytic" or "true a priori" instead of "logico-conceptually true".[3])

(LC) is now generally rejected. Being logico-conceptually true is sufficient, but not necessary, for being necessarily true. In effect, we've already seen why. Any logico-conceptual truth is a priori. But not all necessary truths are a priori. *Water is* H_2O and *gold is the element with atomic number 78* are necessary, but not a priori, and thus not logico-conceptually true.

There is another reason to reject (LC). For a proposition to be logico-conceptually true is for its negation to *entail* a contradiction (a proposition of the form [P and not-P]). Obviously the notion of necessity is embedded in the notion of entailment: P entails Q if, roughly, the truth of Q is conceptually necessary given the truth of P. So (LC) gives a circular

[3] So, in this paper, I will use the terms "logico-conceptual" and "analytic" and "a priori" more or less interchangeably. Of course, a case can be made that *there are* important distinctions among the meanings of these terms. (For example, "analytic" is typically, though not always, used as a predicate of *sentences*, not propositions.) But none of these will have any relevance in the present inquiry. The only important distinction will be between propositions like *triangles have three sides*, on the one hand, and *water is* H_2O, on the other. The latter is a posteriori: it is *not* such that a grasp of the concepts involved is sufficient to decide its truth or falsity. The former *is* such that a grasp of the concepts involved is enough to determine its truth or falsity. In this paper, I will use different terms to characterize propositions like the former: "analytic", "logico-conceptually true", "true a priori".

ceptually necessary given the truth of P. So (LC) gives a circular analysis of necessity.[4] In this paper, I will take it for granted that (LC) is false, for the reasons just stated.[5]

A doctrine sometimes known as *possible world semantics* is now widely taken to give the truth about necessity and possibility. Pws is, in its essentials, given by the following contentions:

(a) A proposition is a function from worlds to truth-values (or is a set of worlds).
(b) A proposition is necessarily true iff it is true in all possible worlds
(c) A proposition is possibly true iff it is true in some possible world.

(b) and (c) are relatively clear. But (a) may require clarification. The import of (a) is that *snow is white* can be analyzed in one of two ways. On the one hand, it can be seen as a function that associates the truth-value *true* with worlds where snow is white and associates the truth-value *false* with worlds where snow is not white. On the other hand, it can be seen as the set of worlds where snow is white. The idea is that, if you know for some proposition exactly what circumstances or "worlds" would make it true and exactly what circumstances would make it false, then you know everything there is to know about that proposition. So if you know, for any possible world, what *truth-value* a proposition assigns to that world, then there isn't anything left for you to know about that proposition. Equivalently, if you know exactly which worlds fall into the set of worlds to which a proposition assigns the truth-value *true*, and which worlds do not, then you know everything there is to know about that proposition. Given this, we might as well just *identify* the proposition with an assignment of truth-values to worlds, or with the set of worlds to which the proposition assigns the truth-value true. (I myself do not think that propositions are to be thought of this way or that this reasoning is good; I am simply stating the theory.)

I have two objectives in this paper. First, I will set forth some reasons to reject pws. Second, I will set forth a positive account of what necessity and possibility consist in.

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[4] Blackburn 1993 (chapter 7) makes this point, attributing it to Quine 1990/1951. Long ago, Pap 1958 (chapter 1) made this point.

[5] This discovery is due, of course, to Saul Kripke 1972 and also, to some extent, to Hilary Putnam 1975. If I am not mistaken, it was anticipated by Pap 1958 (chapter 11).

By way of anticipation, my positive account of modality will be this. A proposition is necessarily true iff it is made true entirely by facts about the *decomposition of properties.* Consider the proposition *anything over 7ft tall is over 6ft tall.* This is a necessary proposition, if ever there was one. Why is it true? Because for something to be over 7ft tall *is* (inter alia) for it to be over 6ft tall. So the property of being over 7ft tall decomposes into (inter alia) the property of being over 6ft tall.

There are, of course, various problems with this account. It isn't immediately obvious how it applies to truths like *if P, then P or Q* or *2+2=4.* Further, that analysis might seem to be circular (throughout this paper, statements in italics should be taken to be those of an imaginary critic):

Properties are abstract objects and therefore do not in any literal sense "decompose"; a property cannot be decomposed in the sense in which a chair can be decomposed. So when you say that the property of being over 7ft tall "decomposes'" into that of being over 6ft tall, the only clear meaning that can be attached to your statement is this – anything over 7ft tall is necessarily *over 6ft tall. But then your analysis is circular.*

We will later see that these objections turn out to have less force than they seem to at first.

II. First of all, I don't deny that a proposition P is necessarily true iff there is no possible world in which it would be false, and I don't deny the corresponding claim about possibility. There is no doubt that the following biconditional holds: P is necessarily true just in case P holds in every possible world. What I deny is that this statement provides any kind of *analysis* of the notion of necessity. (B) is just an innocuous platitude, which serves as a neutral core around which different, and incompatible, theories of modality may be constructed.

There are a number of problems with pws. As I see it, the most basic problem with pws lies in the fact that it mistakes a *symptom* of necessity for necessity itself.

Consider the proposition *triangles have three sides.* This is a paradigm case of a proposition that would hold "in any world". It is legitimate to ask *why* this proposition holds in any possible world. It is surely not a brute fact that *triangles have three sides* holds in every possible world. It is not as though triangles just happen to have three sides in this world *and* in that world, and so on for non-denumerably other worlds.

Surely *triangles have three sides* holds in all worlds because of something about the structure, the internal architecture, of that proposition – because of something about concepts that compose that proposition and of the way in which they compose it. The fact that this proposition holds in all possible worlds is merely a symptom of this fact about its internal architecture.

This point has epistemological support. Given a necessary proposition, how do we *know* that it is necessary? How do we *know* that *triangles have three sides* is necessary? Obviously we *don't* know this by taking a tour of all the non-denumerably different possible worlds. We know it by inspecting the proposition itself. Whatever it is that makes the proposition be necessary, we can grasp that something in its entirety without leaving this world.

There are more formal reasons for rejecting pws. First of all, as it stands, pws is circular: (c) is obviously circular – it defines possibility in terms of itself. And so, by implication, is (b).

There is an obvious way to make (c) be non-circular. We should replace

(c) A proposition is possible iff it is true in some possible world

with

(c') A proposition is possible iff it is true in some world.

We must drop the "possible" from the *definiens* in (c).

The same point applies to (b). As it stands, (b) is circular, if only implicitly. The proposition

(b) A proposition is necessary iff it is true in all possible worlds

is equivalent to

(**b)** A proposition is not possibly not true iff it is true in all possible worlds.

Obviously **(b)** is circular. So we must drop the "possible" from the *definiens* in **(b).** This leaves us with:

(b') A proposition is not possibly not true if it is true in all worlds

or simply

(b') A proposition is necessary iff it is true in all worlds.

(Everything we henceforth say about (b') applies *mutatis mutandis* to (c'). So to avoid verbosity, I will just talk about (b'), and leave implicit the associated points about (c').)

The question arises: what is meant by the expression "some world" in (b')? Trivially, the word "world" in (b') either denotes something concrete – a space-time manifold comprising causally efficacious objects – or it does not. In other words, (b') can be read "actualistically" or "non-actualistically". I now wish to show that, on either reading, (b') is untenable.

First let us consider the actualistic reading: a proposition is possible iff it is true in some world, where the term "world" refers to something concrete.

In that case, a well known problem arises. If necessity is identical with truth in all worlds, where "world" denotes something concrete, then how could we possibly *know* that *triangles have three sides* is necessary?[6]

Obviously we cannot take a tour of different worlds. And even if we could, we still could not, on the basis of such a tour, establish that *all triangles have three sides* was necessary.

For the sake of argument, suppose that you could hop from world to world, and that, consequently, you *could* travel to other worlds to see if *triangles have three sides* was true over there. (What I am saying about *triangles have three sides* will apply to any necessary proposition.) Further, suppose for the sake of argument that to establish the necessity of *all triangles have three sides*, you *did* have by going to other worlds and see if it held in those worlds.

There are infinitely many – indeed, non-denumerably many – different ways the world might have been. (There are non-denumerably many points in space. A given particle could occupy non-denumerably points *other* than the one it does in fact occupy. Therefore, there are non-denumerably many ways the world might have been.) So, if there is a world corresponding to each possibility, then there are non-denumerably many worlds. So *if* you

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[6] See Peacocke 1999 (chapter 1).

had to establish the necessity of *all triangles have three sides* by touring different worlds, you'd have to go to non-denumerably many worlds.

But even if (what would be doubly impossible) you visited each of these non-denumerably many worlds, that would *still* not suffice to establish that *triangles have three sides* is necessary. For you'd need some guarantee that you had visited *every* different world; you'd need some guarantee that you had considered a world corresponding to *each* possibility. Even if you had visited every world, unless you *knew* that you had done so, you could not conclude from what you found on your tour that *triangles have three sides* was true in *every* world. And in order to *know* that you'd visited a world corresponding to each possibility, you would *already* have to know what was possible and what wasn't. But if you already knew this, then the tour of the worlds would be superfluous. So you *couldn't* learn that *triangles have three sides* is necessary by taking a tour of worlds unless you *already* had some way of knowing what was possible and what was not. But if you had that knowledge, then you would *already* know that *triangles have three sides* is not possible.

In sum, a tour of possible worlds can tell you only what you already know, at least as far as modal facts are concerned. So we know that *triangles have three sides* is necessary independently of any such tour.

This line of thought surely gives us reason to read (c') non-actualistically. If we treat worlds as concrete objects, in the same category as our own world, then we distort the way in which we know the modal properties of propositions. So to avoid that distortion, we must treat alternative "worlds" as non-concrete objects.

Now if an alternative world is not to be seen as something concrete, then how *is* it to be seen? The obvious answer is this: an alternative world must be seen either as a description of some kind, and thus as a set of propositions, or as a model of some kind.

Let us consider each of these options. First, let us consider the idea that an alternative "world" is a set of propositions. In that case, (a) becomes:

(a_p) A proposition is a function from sets of propositions to truth-values (or is a set of sets of propositions).

(a_p) is viciously circular. In any case, (a_p) provides no good analysis of what a proposition is.

What becomes of (b) and (c) if worlds are treated as sets of propositions? (b) becomes:

(b_p) A proposition is necessarily true iff it is true in all sets of propositions.

And (c) becomes:

(c_p) A proposition is possibly true iff it is true in some set of propositions.

Let us focus on (c_p) for a moment; what we will say will apply *mutatis mutandis* to (b_p). What would it be for a proposition P to be true "in" some set of propositions? As far as I can tell, the only reasonable answer to this question is this: A proposition P is true "in" some set of propositions [P_1, $P_2 \ldots P_n$] iff, in order for all of P_1, $P_2 \ldots P_n$ to be true, P must be true. Basically, a proposition P is true "in" some set of propositions if the truth of P is requisite to the truth of the members of that set.

So the proposition *grass is green* is true "in" the set of propositions [*Socrates is tall, snow is blue, grass is green*] because in order for all the members of that set to be true, it is necessary that *grass is green* be true. And *something is green* is true "in" the set of propositions [*Socrates is tall, snow is blue, grass is green*] because, even though arguably that proposition is not itself a member of that set, its truth is prerequisite to the joint truth of the members of that set.

With this point in place, let us evaluate (c_p). It is immediately obvious that (c_p) provides an analysis of possibility that is simply wrong. For *every* proposition is such that its truth is requisite to the truth of the members of *some* set of propositions. Consider the proposition *water is not H_2O.* Let S' be the set [*grass is green, roses are red, water is not H_2O*]. Obviously the truth of *water is not H_2O* is requisite to the truth of the members of S'.

Since (c_p) provides a wrong analysis of possibility, it follows by implication that (b_p) provides a wrong analysis of necessity. This follows because necessity and possibility are interdefinable.

In sum, if alternative worlds are treated as sets of propositions, then pws becomes both circular and false.

What about the idea that alternative worlds are models? Does this enable (c') to capture the nature of necessity? (And does it enable (b') to capture the nature of possibility?) It does not.

To begin with, if alternative worlds are models, then (c') becomes:

(c'_m) A proposition is possibly true iff it is true in some model.

The problem is that there are both possible and impossible models: models which describe possible states of affairs, and models which do not. So there are models in which *water is not H_2O* and *Socrates is not self-identical* are true. So as it stands, (c') is simply false: for *any* proposition is true in some model. *Socrates is not self-identical* is true in a model (albeit an impossible model). But that proposition is not possibly true.

Thus (c'_m) must be restricted; to make it avoid being false, we must convert it to:

(c''_m) A proposition is possibly true iff it is true in some possible model.

But (c''_m) is obviously circular. By implication, (b') becomes comparably circular if we treat worlds as models.

Also, a model just *is* a set of propositions, in the final analysis. So with regard to the points we made in connection with the idea that worlds are sets of propositions, those points also apply to the idea that alternative worlds are models.

III. We've seen reason to reject the possible worlds analysis of necessity. So what is the right analysis? We must remember what Hume said: a necessary relation never holds between two completely distinct things. Necessity is always grounded in identity or inclusion. Given any two distinct things – say, my desk and my computer -- nothing about the one *necessitates* anything about the other. But the story is different where non-disjoint things are concerned. If a *part* of my computer is changed, that *does* necessarily affect the computer as a whole: if the mass of *part* of the computer is diminished, that (ceteris paribus) necessarily reduces the mass of the computer as a whole. So it is not unreasonable to look for the grounds of necessity in some kind of identity or inclusion-relation.

The problem is that not just any kind of inclusion relation grounds a necessary truth. Let C be some particular cell that my body happens to include. My body need not contain C; it is a contingent fact that it contains C. Suppose that so and so is in the army. Surely so and so didn't *have* to join the army, i.e. he didn't *have* to be a part of the army. In general, if x is a part of y, where x and y are spatio-temporal objects, that fact may hold contingently. So that relation *by itself* doesn't ground necessity.

This point holds *mutatis mutandis* of events. World War II can be thought of as an extremely long, complex event. But surely not every one of the sub-events composing World War II *had* to be a part of it; surely it is not a necessary fact about World War II that private so and so fired two rounds as opposed to one round in some particular battle. Thus inclusion relations holding among spatio-temporal entities, whether objects or events are often contingent. So *by itself* spatio-temporal inclusion doesn't ground necessary truth.

A more interesting proposal is this: Necessary truths are grounded membership in sets. Let S be the set [a,b,c]. Surely S necessarily contains a; if something fails to contain a, then *ipso facto* it is not S. For what it is to be S just *is* (inter alia) to contain a. Maybe all necessity can be analyzed on this model.

I do not myself think that necessity can always be analyzed in terms of set-inclusion. But before I say why, I'd like to respond to some *erroneous* reasons to dismiss the analysis in question (this will help motivate the positive analysis we will give later on):

Sets often only contingently *contain their members. Consider the set of humans. This includes George Bush. But it doesn't necessarily contain him; for he might not have been born. So* by itself *membership in a set does not ground necessity.*

I think that the property of being a human determines *different* sets; when somebody dies, any set previously containing that entity simply *ceases* to exist. So the set of humans – that is, the set containing myself, you, George Bush, and so on – *is* modally frozen; it *does* necessarily contain George Bush. In general, sets *are* modally frozen. And when they appear not to be, that is really because some one property, e.g. the property of being human, generates different sets at different times.

Here is another erroneous reason to reject the analysis under consideration:

There is no way to analyze the necessity of if P, then P or Q *or* P or not P *in terms of inclusion in a set.*

This isn't true. *If P, then P or Q* is plausibly seen as saying: *the set of possible truth-makers of P is included in that of [P or Q]*. And *[P or not P]* is plausibly seen as saying *any possible truth-maker falls either in the set of truth-makers of P or in the set of truth-makers of not-P.*

Nonetheless, it is quite clear that necessity cannot typically be seen as holding in virtue of facts about set-membership. There are two reasons. First, any attempt to identify the relevant sets will often make use of the notion of possibility (and therefore of necessity: possibility being definable in terms of necessity). Second – what is related -- in many cases, the sets in question would contain *infinitely* many members. So there would be no way to identify these memberships except in terms of some *rule*. And embedded in the concept of a rule is the concept of necessity. Thus the attempt to reduce necessity to set-membership ends up being circular.

Once again consider the proposition *if P, then P or Q.* As we just noted, this can be seen as holding in virtue of some containment relation among sets: the set of possible truth-makers of P is included in the set of possible truth-makers of [P or Q]. But here we are defining "necessity" in terms of sets of "possible" truth-makers; so we are, by implication, defining "necessary" in terms of itself. Surely, if we are to account for the necessity of *if P then P or Q* in terms of facts about membership in sets of truth-makers, we must talk about *possible* and not merely *actual* truth-makers. If we take *if P, then P or Q* to say something about sets of truth-makers, it must be this: for any propositions P and Q, the set of *possible* truth-makers of P is included in the set of *possible* truth-makers of [P or Q]. Propositions are individuated, not by what their *actual* truth-makers are, but by what their *possible* truth-makers are. "The inventor of bifocals was smart" and "the first-postmaster general was smart" are *made true* by the same thing, viz. that Benjamin Franklin was smart. But they are different propositions, because they have different *possible* truth-makers. (If Newton had invented bifocals, and Gauss had been the first-postmaster general, then those two propositions would have different truth-makers.) *If P, then P or Q* obviously makes a statement about propositions. Propositions, if understood in terms of sets of truth-makers, must be understood in terms of *possible*, and not merely actual, truth-makers. Therefore, if we construe *if P, then P or Q* as saying something about sets of truth-makers, we must take it as saying something about sets of *possible*, not merely actual, truth-makers. But then, in our attempt to account for the necessary status of *if P, then P or Q*, we end up defining "necessary" in terms of "possible", and therefore in terms of itself.

Also, there may be infinitely many possible truth-makers of P and of Q. This will be the case for most values of P and Q. There are infinitely many different possible states of affairs that would make true the proposition *Bob is over 6 ft tall* – Bob could be 6 ft, one inch or 6 ft, 1.1 inches, or 6 ft, 1.11 inches, and so on. I simply don't see how the set of possible truth-makers of *Bob is over 6ft tall* could be identified except through some rule and, therefore, without using the concept of necessity.

To sum up, it won't do to see necessity as *typically* being grounded in facts about set-membership. For often such facts themselves involve the concept of necessity. (I think that *in some cases* necessity reduces to facts about set-membership – in cases where the sets in question are finite. And I think that this fact points the way to a correct understanding of necessity: that is why I have made such heavy weather of it.)

Here are the facts, as I see them. First, necessity must be grounded in some kind inclusion relation (identity being a limiting or degenerate case of inclusion). Second, the right kind of inclusion is not (typically) mereological inclusion in some spatio-temporal object or event. Third, the right kind of inclusion is not (typically) inclusion in a *set.*

There is a fourth point. The concept of necessity is not a disjunctive one. Any correct analysis of necessity must *not* be of the form "a proposition is necessary if it holds *either* in virtue of such and such *or* in virtue of thus and such…" So it would be no analysis of necessity to say: "a proposition is necessary if it is made true *either* by some fact about membership in sets *or* by some fact about the constitution of a physical object *or*…." Some *one* thing must ground necessity in all cases. In the next section, I will try to say what this one thing is.

IV. Now I will outline a positive analysis of necessity. Propositions, even contingent ones, can be seen as holding in virtue of facts about concepts. *John is tall*, though contingent, is equivalent to some fact about concepts: *the concept* John *is uniquely instantiated and any instance of it is tall.*

Now concepts pick out properties. The concept

(*) *shape whose surface coincides with the class of all points equidistant from a given point in a 3-D space*

is different from the concept

(**) *closed 3-D figure of uniform curvature.*

But these concepts pick out *the same property*: that property had by basketballs, tennis-balls, planets, and so on --- the property of being spherical.[7] It seems to me, also, that (*) and (**) are necessarily co-extensive *because* they pick out the same property.

Concepts, it seems, can be thought of as modes of presentation of properties. The property of sphericity is presented by (*) and also by (**): (*) and (**) are modes of presentation of that property. There can be no doubt that there are different ways of thinking about properties; and concepts are such ways.

Also, any property can be given by *infinitely* many different concepts. Consider the property of duality (i.e. the property had by a set S such that for some x and some y distinct from x, x and y belong to S and for any z, z belongs to S iff z is identical with x or y). There are infinitely many ways to think about that property (*1+1, 4÷2….*) Even a property like that of being green can be accessed in infinitely many ways. Let C be some concept of that property, i.e. some way of thinking about that property (surely there

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[7] No doubt, some will say that there is a one-one correspondence between properties and concepts and that, being different concepts, *closed 3-D figure of uniform curvature* and *shape whose surface coincides with the class of all points equidistant from a given point in a 3-d space* pick out different properties, albeit necessarily coextensive ones. I cannot fully address this point of view here. But it seems to me untenable. Incontestably, there is *one* shape associated with those two concepts: it would not be possible to create an object that had the shape picked out by the one concept but didn't have the shape picked out by the other; it would be false – strictly and mathematically false – to say that those two concepts picked out different *shapes.* Now obviously each of those concepts picks out a shape-property: a property that an object has wholly in virtue of having a certain shape. So those concepts pick out the same property, even though those concepts are different from each other.

Exactly analogous remarks show that 4÷2, 2, 976 minus 974, and so on, all pick out the same *property* The concepts 4÷2, 2, 976 minus 974, and so on, are all different. Surely "Joe has 976 minus 974 apples" has a different "sense" from "Joe has 4÷2 apples", and that difference in sense is obviously due to the fact *that 976 minus 974* corresponds to a different mode of presentation from *4÷2*. At the same time, all of those concepts pick out the same property -- that property had by a set S such that for some x and some y, x and y belong to S, x is distinct from y, and nothing z belongs to S that isn't identical with either x or y. For there is absolutely no fact that could make true "Joe has 976 minus 974 apples" without also making true (e.g.) "Joe has 24÷12 apples" and *vice versa*. It is a strict mathematical fact that the number of apples picked out by *976 minus 974* is the same as that picked out by *24÷12*. So those concepts pick out the same number-property (in this case a property possessed by the set *apples belonging to Joe*). So the identification of concepts with properties is not tenable, and the view that concepts are modes of presentation of properties is *de rigueur*.

is such a concept). In that case, x is green if x falls under C or x falls under [*C or square circle*] or x falls under [*C or regular seven sided figure*], and so on. To sum up, concepts are ways of thinking about properties, and a given property can be thought about through different concepts.

Given this, consider the statement *triangles are closed figures.* Why is this is necessarily true? For something to have the property of being a triangle *is* (inter alia) for it to have the property of being a closed figure. The property of being triangular includes the property of being closed. We might also say that the former "is constituted by" or "decomposes into" the latter property.

Consider the statement *anything over 7ft tall is over 6ft tall.* This is true because for something to be over 7ft tall just *is* (inter alia) for it to be over 6ft tall. The former property includes the latter.

Consider the proposition *water contains hydrogen.* (Unlike *triangles are closed* this is *a posteriori.*) This is necessarily true because for something to be water just *is* (inter alia) for it to contain hydrogen. The property of being water includes the property of containing hydrogen.

I think it is worth investigating the idea that necessarily true statements are all made true by relations of inclusion among properties, taking identity as a degenerate or limiting case of inclusion.

Perhaps the following will give the rudiments of a theoretical basis for this line of thought. As we noted a moment ago, any proposition can be seen as being wholly about *concepts.* This is obviously true of statements like *squares are closed figures* (anything falling under the concept *square* falls under the concept *closed figure*). But it is true even of contingent, object-involving statements like *Bob is wet* (*the concept* Bob *is uniquely instantiated and anything instantiating that concept instantiates the concept* wet).[8] So if a statement is true, that is always in virtue of some fact about *concepts.*

Therefore if a statement is *necessary* that is due to some fact about concepts. For *squares are not round* to be necessary is for the proposition *necessarily, squares are not round* to be true. So the necessity of the former is

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[8] To be sure, the property of being instantiated may not be a *necessary* property of the concept *Bob.* Not all properties had by concepts are *necessary.* One might argue that 1000 years from now, the concept *Bob* will no longer be instantiated, just as the concept *Socrates* is not currently instantiated, though it used to be. I myself am open as to whether this is good reasoning or not. In any case, it is clear that concepts can have *some* of their properties only contingently. The concept *electronic device on my desk* has the property of being instantiated by one object – my computer – but obviously it is a *contingent* fact about that concept that it is instantiated by exactly one object.

the truth of the latter. And the latter, like any proposition, holds entirely in virtue of facts about concepts.

Thus it is reasonable to look for the basis of necessity in some kind fact about concepts and their interrelations. The obvious candidate is: Some kind of *entailment* relation must hold. But we now know that to be the wrong answer (*water is H_2O).*

The solution lies in the point about concepts made a moment ago. Concepts are modes of presentation of properties. Even concepts of *objects* – e.g. the concept *Bob* – can be seen as a mode of presentation of a property – the property of being identical with Bob.[9] So given a necessary proposition P, it is natural to look for the basis of P's being necessary in some kind of fact about the *properties* represented by the concepts constitutive of P: presumably some fact about their interrelations. *Squares are closed figures* is necessarily true because what it is to have the property of being a square is (inter alia) to have the property of being a closed figure; the one property is inclusive of the other. (And *squares are not closed figures* is necessarily false because *squares are closed figures* is necessarily true.) *Water contains hydrogen* is necessarily true because what it is for something to be water *is* (inter alia) for it to contain hydrogen. The same is true *mutatis mutandis* of *water contains oxygen* – and it starts to look as though our analysis of necessity can account for the necessary status of *water is H_2O.*

So it looks promising to try to ground necessity in some fact about *properties.* And it looks as though the relevant fact has to do with the *decomposition* of properties (being square *decomposes* into being closed) or in facts about *inclusion relations* holding among properties (being square *includes* being closed).

There are some objections that could be made to this analysis. Let us now deal with these:

I simply don't see how your analysis could accommodate some very basic necessary truths, like if P, then P or Q *and* 1+1=2.

According to many, *1+1=2* says that for a collection to contain two objects *is* (inter alia) for it to contain one object x, and *is* (inter alia) for it

[9] It must be noted that, in logic, the grammatical distinction between noun and verb is of no importance. The proposition *x is identical with Bob* is often construed as having the form *Bob(x)* or *x Bobs.* I think that this is actually a more correct way to understand "entities" like Bob. For what is most basic are *instances* of properties; whatever *objects* there are in the world is fixed by what properties are instantiated.

to contain one object y (other than x), and *is* (finally) for it not to contain any third object. So *1+1=2* says that the having one property – being a dual set – decomposes into the having of other properties – that of containing an x, and a y different from x, and that of *not* containing a z not identical with either x or y. If this is the right analysis[10] -- and off-hand it is obviously a reasonable one -- then surely *1+1=2* is no counterexample to our thesis.

Our analysis can also accommodate the necessity of *if P then P or Q*. Let **P** be the property of being such that P is true. (So if P is *grass is green*, then every object has **P**. For, vacuously, every object is such that grass is green.) Let **Q** be the property of being such Q is true. (So if Q is the proposition *snow is purple*, then no object has **Q**.) The proposition *if P, then P or Q* is only trivially different from the proposition *for all x, if x has **P**, then x has **P or Q**.*

Now the property of having **P** is not something altogether different from the property of having **P or Q.** Having **P** is *a way* (not the only way) of having **P or Q**. For something to have **P** *is* (inter alia) for it to have **P or Q** – having **P** *consists in*, among other things, having **P or Q**. So *for all x, x has **P**, then x has **P or Q*** is made true some inclusion relation holding among properties.

This can be thought of as follows. Being in Paris is not something *different* from being in France. Being in Paris is a way of being in France. To be in Paris *is* (inter alia) to be in France. So the property of being in Paris *includes* the property of being in France. Similarly, the property of having **P** *includes* the property of having **P or Q.** To tie this in to our analogy: being in the region of property-space corresponding to ***P*** is not *different* from being in the region of property-space associated with ***P or Q***. Being in the former property-space is *a* way of being in the latter, just as being in Paris is *a* way of being in France. Given that having **P** is a way of having **P or Q**, it seems fair to say that having **P** *includes* or *involves* having **P or Q** , and thus decomposes into (inter alia) having ***P or Q.*** Now the truth of *if P then P or Q* can be represented as holding in virtue of the just mentioned facts about **P** and **P or Q**, and thus in virtue of a fact about the decomposition of **P.**

Let us deal with another criticism:

[10] There are different reconstructions of arithmetic – the one given here is basically the Fregean reconstruction. But what we said about that reconstruction would apply *mutatis mutandis* to any other, e.g. the Neumannian one.

Your analysis is circular. To say that the property of being over 7ft tall includes *the property of being over 6ft tall is to say (inter alia) that anything over 7ft tall is* necessarily *over 6ft tall. Your talk of property inclusion is merely obscure talk of necessity. So your analysis is circular.*

I do not think this criticism is just. It is easy to make the case that facts about property inclusion are *more basic* than facts about necessity, and that, consequently, talk of necessity reduces to talk of property-inclusion, but not *vice versa.* So talk of property-inclusion *underlies* talk of necessity, and can thus provide a non-circular analysis of necessity.

As we noted earlier, it is surely no *accident* that anything, in any possible circumstance (world), that is over 7ft tall is over 6ft tall. It is not as though, in every possible circumstance, the things that have the first property just happen to have the second. Obviously this fact is grounded in *what it is* to be over 7ft tall. So the necessity is *grounded* in something, and this something is surely a fact about what it is to over 7ft tall. And there doesn't seem to any way of capturing this grounding-relation except by saying that part of being over 7ft tall *is* being over 6ft tall.

To sum up, the circularity-charge would have validity only if the case could be made that talk of necessity somehow provided the foundation for talk of property-inclusion. But this case cannot be made. It is quite clear that the necessity of *anything over 7ft tall is over 6ft tall* is grounded in some fact about the property of being over 7ft tall. And the relevant fact would seem to be that being over 7ft is somehow inclusive of being over 6ft tall.

Surely what it is for something to be red or square couldn't possibly vary from circumstance to circumstance or, if I may so speak, from world to world.[11] In other words, what it is for something to be red or square is necessarily the same thing in different circumstances. This necessity is easily *grounded* – is easily traced to something deeper. Properties are not con-

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[11] I think it is highly *useful* to speak of possible worlds; it is very *useful* to say things like "there is no possible world where squares are round". But I deny that any *analysis* of possibility/necessity in terms of the notion of a possible world – if only for the reasons pointed out earlier. 'P necessary iff P is true in some possible world' is blatantly circular, since "possible" just means "not necessarily not". And if drop the "possible" from the *definiens*, and commit ourselves to actualism, we run into serious epistemological problems. Not to mention the problem that, if a proposition is true in all possible worlds, that fact is surely *grounded* in some fact about the proposition, which fact ought surely to be considered the essence of necessity: truth in all possible worlds being a mere symptom of necessity.

stituents of circumstances (they are platonic objects). So they don't fall within the clutches of circumstance. Since properties are not *in* circumstances or worlds, it follows (vacuously) that they don't vary *from* circumstance to circumstance. For x to vary *from* circumstance to circumstance is just for x to meet two conditions: (i) x must be *in* different circumstances and (ii) x must be different in different circumstances. So the circumstance-invariance of properties reduces to the fact that properties are not *in* circumstances at all.[12]

Thus what is *included in* being square is necessarily the same from circumstance to circumstance. And *this* necessity is easily grounded. Since properties are not to any degree in circumstances, they don't vary from circumstance to circumstance; *a fortiori* what is *included* in a property doesn't vary from circumstance to circumstance.

The point is this discourse is to show that our analysis doesn't involve any kind of covert circularity. It is a true fact that properties are *necessarily* the same from circumstance to circumstance, and that, consequently, what is included in properties (and thus in having them) is necessarily the same from circumstance to circumstance. But this necessity is easily reduced to some non-modal or sub-modal fact. So although it is true that our analysis makes use of the fact that properties necessarily don't change from circumstance to circumstance, this does not mean that our analysis is circular; for we have reduced that necessity to a sub-modal fact.

What I've just said may not satisfy everybody:

Admittedly, for something to be over 7ft tall is *(inter alia) for it to be over 6ft tall. But that fact is* itself *to be traced to some necessity, viz.* anything over 7ft tall is necessarily over 6ft tall. *So inclusion relations among properties are to be explained in terms of necessities, and therefore cannot themselves explain necessities.*

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[12] It may be said that my analysis of necessity makes use of the entailment: *if x isn't in any circumstances, then it cannot be both (i) in different circumstances and (ii) be different in different circumstances.* So, in as much as my analysis uses this entailment, it uses the concept of necessity; for to affirm an entailment is just to affirm a certain kind of necessity. So my analysis is circular.

But this entailment reduces to an inclusion relation – just as my analysis predicts! It reduces to the fact that condition (i) is included in the joint condition [(i) and (ii)]. In other words, it reduces to the fact that satisfaction of [(i) and (ii)] *is* (inter alia) satisfaction of (i.).

This is just the point that I am trying to combat. Facts about inclusion relations among properties do *not* have to be explained in terms of necessities. In fact, they *are* not to be so explained. Such facts are to be "explained" in terms of the banal point that properties are not *in* circumstances at all and therefore don't vary from circumstance to circumstance. [13]

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[13] The contents of this footnote are similar to those of the last footnote. But it is worth repeating myself to stave off the unjust – but inevitable – accusation that our analysis of necessity is circular.

The argument just given makes use of an entailment, namely:

Properties are not in circumstances; therefore properties don't vary from circumstance to circumstance.

And in as much as an entailment is a kind of necessity, and in as much as my analysis uses that entailment, my analysis might seem to be guilty of circularity. That is not the case. Our analysis is not circular. The entailment/necessity in question reduces to a case of property inclusion, just as my analysis predicts. For something to vary from circumstance to circumstance just *is* for it (i) to be *in* different circumstances and (ii) for it to be *different* in different circumstances. So the entailment

"properties are not in circumstances"

to

"therefore properties don't vary *from* circumstance to circumstance"

is identical with the entailment from:

(a) 'Properties are not (i) in circumstances'

to

(b) Properties are not *both* (i) in circumstances and (ii) different in different circumstance.

And the entailment from (a) to (b) – i.e. the fact that (b) is *conceptually necessary* given (a) – is easily reduced to a deeper fact, one which can be delineated with using the concept of necessity. That deeper fact is that the property of satisfying (i) is *included* in the property of satisfying [(i) and (ii)]. And this is just as my analysis predicts. So there is no covert circularity in our analysis. In fact, when we investigate the allegation of some such circularity, our analysis turns out to be vindicated.

There is an obvious counter-example to your analysis. Suppose that Bob's favorite concept is square. *Now the statement*

(A) anything that falls under the concept *square* is a closed figure

is necessarily true. On the other hand, the statement

(B) anything that falls under Bob's favorite concept is a closed figure

is, while true, not necessarily *true (for Bob's favorite concept might have been the concept* open figure*). But the property-relations – the inclusion-relations among properties -- which make (A) true are the same as those which make (B) true. Both are made true by the fact that, included in the property of being a square, is the property of being closed. So with regard to the statement*: anything that falls under the concept *square* falls under the concept *closed figure – whatever it is that makes that statement be necessarily true, it is not* merely *inclusion relations among properties. For the exact same inclusion relations are associated with the contingent proposition:* anything that falls under Bob's favorite concept is a closed figure*.*

This argument involves a fallacy. The property of being a square *does* include the property of being a closed figure. The property of being Bob's favorite concept *does not* include the property of being a closed figure. The property of being Bob's favorite concept is (unlike the concept *square*) something completely different from the concept *closed figure.* There is no inclusion relation there. So my analysis correctly predicts that (B) is contingent.

Now it is true that, in actual fact, the thing which *has* the property of being Bob's favorite concept – this thing being the concept *square – is* such that anything falling under it necessarily has four-sides. But that is irrelevant. (B) relates the property of being closed to the property of being Bob's favorite concept, not to the property of being a square. (A) relates the property of being closed to the property of being square. There is an inclusion relation here. So the proposition is necessary (as our analysis predicts). On the other hand, (B) relates the property of being a closed figure to the property of being Bob's favorite property. There is no inclusion relation here. So the proposition is contingent, just as our analysis predicts.

It is true that the property of being a square is identical with Bob's favorite property. But the property of being a square is not identical with the property of being Bob's favorite property. (B) relates the property of being closed to *the property of being Bob's favorite property*, which is *not* the same thing as the property *square*. And no inclusion relation holds between the property of being closed and the property of being Bob's favorite property. So our analysis predicts that (B) is contingent, and in fact (B) is contingent. (A), on the other hand, relates the property of being closed to the property of being a square. There is an inclusion relation here. So our analysis predicts that (A) is necessary, and (A) is necessary.

These points may help us dispose of another, very similar objection:

Consider the proposition

(*) the first post-master general was identical with the inventor of bifocals.

This is obviously contingently true. But the concepts in that sentence – first post-master general *and* inventor of bifocals – *pick out the same property, the property of being Benjamin Franklin. So those concepts pick out properties that stand in just the kind of relation of inclusion (taking identity as a limiting case of inclusion) that, according to your analysis, grounds necessity. So your analysis predicts that (*) will be necessary. It isn't. So your analysis is wrong.*

The problem here is obvious. The *property* of being the inventor of bifocals is totally different from the *property* of being the first post-master general. And the expressions "the first post-master general" and "the inventor of bifocals" don't pick out the same *property*: they pick out the same *individual.* They pick out Benjamin Franklin, but not the *property* of being Benjamin Franklin. (*) doesn't say that the property of being Benjamin Franklin is included in, or includes, the property of being Benjamin Franklin. (That would be true enough. But it is not what (*) says.) Since the property of being the first post-master general is totally disjoint from the property of being the inventor of bifocals, our analysis predicts that (*) will be contingent. And that is in fact the case.

(*) is not made true by facts about the decomposition of properties; it is made true by the fact that some one individual had two quite distinct sets of properties. The proposition

(**) *anything that is a square is closed figure* is necessarily true because, specifically, it is made true by a fact about what is included in having some one property. So it is made true by some fact about the decomposition of some property: to have the property of being square-shaped *is* or *includes* (inter alia) having the property of being a closed figure. Since (**) is made true entirely by some fact about what is included in having a certain property, our analysis predicts that (**) will be necessary; and this prediction is correct.

I would like to end this section by considering one last possible misgiving about our analysis:

Off-hand, I don't see how your analysis accommodates the necessity of "if Mary is taller than George, then George is less tall than Mary".[14]

Remember what we said earlier. The proposition *x is water* is obviously different from the proposition *x is* H_2O. But we were able to account for the necessity of *water is* H_2O and, therewith, for the necessity of *if x is water, then x is* H_2O *(and vice versa).* We said: the property of being water includes the property of being composed of H_2O (and vice versa). What it is for something to be water *is* for it to consist of hydrogen atoms and oxygen atoms arranged in certain ways.

Surely what it is to have the property of being a thing x such that x is less tall than Mary is not something different from what it is to have the property of being a thing x such that Mary is taller than x. It is not as though we are dealing with two properties here, any more than we are dealing with two properties in the case of *water* and H_2O. The *concept* of being a thing x such that Mary is taller than x may be different from the *concept* of being a thing x such that x is less tall than Mary, just as the concept *water* is different from the concept H_2O. But the *property* of being a thing x such that Mary is taller than x is not different from the property of being a thing x such that x is less tall than Mary, just as the *property* of being water is not different from the *property* of being H_2O. So just as our analysis predicts, the necessity of *if Mary is taller than George, then George is less tall than Mary* is grounded in a property-identity (property-identity being a limiting case of property-inclusion). For, modulo irrelevant facts about verbal packaging, *if Mary is taller than George, then George is less tall than Mary* is surely the same proposition as *if George is a thing x such that Mary is taller than x, then George is a thing x such that x is less tall than*

[14] This point was made to me, virtually verbatim, by an anonymous reviewer at *Metaphysica.*

Mary. And, as we've just seen, the property of being a thing x such that Mary is taller than x is identical with (and is thus, in a limiting sense, inclusive of) the property of being a thing x such that x is less tall than Mary.

V. I would like to give some historical context to the position advocated here.[15] I'm going to be extremely brief, since much of what I'm about to say has been said more thoroughly elsewhere.[16]

I am by no means the first to believe that necessary relations are underwritten by containment- or inclusion-relations. As I mentioned earlier, Hume explicitly said that if x and y are *distinct* – i.e. neither comprises the other and they don't otherwise coincide – then there can be no *necessary* relation between them. Thus, for Hume, any necessary relation between x and y holds in virtue of some containment- or inclusion-relation, identity being a limiting case.

This point is correct as far as it goes. But it is not specific enough to constitute a general analysis of necessity. We've already seen why. How does this point of Hume's explain the necessity of *x is a closed trilateral figure iff x is a figure such that any two of its sides intersect, but not all three of its sides intersect?* The concepts flanking the "iff" are distinct, so it is hard to see how either could really comprise the other. So given *only* Hume's correct point that necessary relations are grounded in relations of non-distinctness, we don't yet have an analysis of necessity.

Actually, Hume does, after a fashion, deal with this last concern. He says that all necessary truths are grounded in "relations of ideas."[17] So *2+2=4* is necessary because it holds entirely in virtue of facts about the relevant *ideas* – the ideas of two and four, and so on. (It is worth investigating how *this* analysis of necessity relates to the other conception of necessity of Hume's that we just mentioned. For reasons of space, we cannot discuss this here.)

But there are some problems with this analysis of necessity. First of all, by "idea" Hume seems to mean "mental image". [18] Presumably, no fact about mental images could make it true that *p and (q or r)* entails *(p and q) or (p and r).* Indeed, not even *recognition* of that truth could consist in a

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[15] The valuable point that my analysis was anticipated by Hume, Kant, and Leibniz was made to me by an anonymous reviewer at *Metaphysica.*

[16] See the chapters on Leibniz, Hume, and Kant in Pap (1958) for much more developed versions of many of the points I will make here.

[17] Hume 1955/1748 (40).

[18] See Pap 1958 (75).

play of mental images. No matter what mental images we might impute to a person, that person's *failing* to recognize that that *p and (q or r)* entails *(p and q) or (p and r)* seems to be consistent with his having those images.

Of course, the obvious reply to make on Hume's behalf is this:

When Hume says that necessary truths are grounded in "relations of ideas", by "ideas" he means concepts *(in the platonic, not the psychological sense). Hume is saying that* 2+2=4 *is necessary because the concepts* two*,* four*, and so on, by themselves guarantee the truth of that proposition.*

This may well be what Hume meant. But in that case his analysis of necessity is counter-exampled by necessary a posteriori truths like *water is* H_2O and *light consists of wavicles.* Also, even if we leave that aside, Hume's analysis of necessity is still not adequate; for he doesn't adequately answer the question *how* the concepts *four* and *two*, and so on, underwrite the necessity of *2+2=4* or *triangles have three sides.*

It isn't, as far as I can tell[19], that Hume has *no* answer to that question, but that he has a false one. He seems to say that *triangles have three sides* is necessary because one cannot imagine a triangle without imagining a three-sided figure. [20] (Hume thus identifies necessity with the property of being such that its negation is *not* imaginable.[21] This identification is a consequence of Hume's strict empiricism: all knowledge is embodied in sense-perceptions and in the "faint copies" they leave , these being mental images. So, for Hume, what is conceivable is what is imaginable. Thus, if we identify possibility with conceivability – and everyone prior to Kripke did so – then we identify possibility with imaginability: a disastrous result, but one that an empiricist has a hard time escaping.) So, in effect, one's image of the one thing *includes* one's image of the other. Thus, Hume has once again anticipated our inclusion-analysis of necessity.

One problem with this view is that there is no way to apply to it truths like *there are more reals than rationals* or even *p and (if p, then either not p or (q and w)), then q and w.* What inclusion- or identity-relations among

[19] I insert this "as far as I can tell" because here I am entering subtleties of Hume-interpretation that are a bit beyond my competence. I am going off of what a reading of Hume, unsupplemented by extensive background reading, would suggest.

[20] Hume 1955/1748 (27-28).

[21] Pap 1958 (81).

mental images could account for so much as our *recognition* of these truths, let alone the truths themselves?[22]

Like Hume, Leibniz anticipated our analysis of necessity. Actually, Leibniz did this in two ways. First, Leibniz said that all necessary truths reduce to the law of identity (*for any x, x=x*). Thus, Leibniz seems to be saying that any necessary truth is grounded in a relation of *identity*. And this is obviously similar to our view that necessity is always grounded in some kind of relation of coincidence or inclusion.

But *what* must be identical with *what*, according to Leibniz, if we are to have a necessarily true proposition? Leibniz's answer seems to be: propositions that are necessarily true hold in virtue of identities of *concepts.* So *bachelors are unmarried* holds in virtue of a *concept*-identity. The concept *bachelor* is identical with a concept like *unmarried adult male.* Obviously the concept *unmarried* is a component of this concept. So *bachelors are unmarried* is true in virtue of an identity holding between the right-side and some component of the left side.

In connection with this, Leibniz held that to *deny* a necessary truth is tantamount to *denying* the law of identity. To say *bachelors are not unmarried* is tantamount to saying *x is not identical with x*, for some value of x.

There are several problems with this analysis of necessity. We will consider two. First, as Pap pointed out, in order to effect the reduction of a truth of logic to anything that could be considered an identity, logical truths *other* than the principle of identity must be used. [23] To use Pap's example, if you want to *show* that as simple a truth as *if p, then p or q* reduces to an identity of any kind, you need to use principles of logic that you must hold *not* to be identical with the law of identity: "[I]n most cases, the reduction of a necessary truth to an identity, total or partial, presupposes principles of deduction which are themselves necessary truths but cannot themselves be held to be in turn thus reducible."[24]

Another problem with Leibniz's view has to do, of course, with Kripke's discovery of necessary a posteriori truths. It is very hard to see how any identity of *concepts* underwrites the truth of *heat is molecular motion.* For the concept *heat* is different from the concept *molecular motion.*

Leibniz anticipates our containment-analysis of necessity in another way. Leibniz notoriously held that *Caesar crossed the Rubicon* is neces-

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[22] See Pap 1958 (75-84) for some related, though not quite coincident points.
[23] Pap 1958 (8-11).
[24] Pap 1958 (8).

sary.[25] Roughly, the idea seems to be this. Caesar is who he is in virtue of the properties that he has. Crossing the Rubicon is one of those properties. So having the property of crossing the Rubicon is *definitive* of being Caesar and thus of the *concept* of Caesar. To be sure, the truth *Caesar crossed the Rubicon* certainly *appears* to be contingent. (Leibniz grants this.) But, according to Leibniz, that is a kind of epistemic epiphenomenon: we are simply ignorant of some facets of the concept of Caesar, just as we are ignorant of some facets of the concept *real number*.[26] If we knew more about the concept *Caesar,* we'd see that, included in it, is the concept *crossed the Rubicon.*

Of course, in actuality, *Caesar crossed Rubicon* is contingent. (We don't really have to time to discuss the errors in Leibniz's argument to the contrary.[27]) But, for our purposes, what is relevant is the conceit that underlies Leibniz's contention that it is necessary. The idea is that the concept *crossing the Rubicon* is somehow *contained* in the concept *Caesar*. In general, containment relations among *concepts* underwrite necessities. So Leibniz has (once again) anticipated our containment-- or inclusion-- analysis of necessity.

In effect, we've already seen the problem with this particular conception of necessity (not surprisingly, it is one that bedeviled many pre-Kripkean views): it doesn't apply to necessary a posteriori propositions. Leibniz says that the concept *crossed the Rubicon* is contained in the concept *Caesar*. Very well. But the concept *consists of hydrogen molecules (inter alia)* is not surely not contained in the concept *water*, even though the proposition *water consists of hydrogen molecules (inter alia)* is necessary.

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[25] He seems to have held that all those propositions that we would characterize as contingent are necessary and, indeed, a priori. Leibniz seems to have held that all propositions are of subject-predicate form (this was generally held before Frege). And he also held 'every predicate, necessary or contingent, past, present or future, is contained in the notion of the subject' (Russell 1992/1900: 27); so *every* proposition is *conceptually* true and thus necessarily true. (Russell 1992/1900 (27)).

[26] See Russell 1992/1900 (chapter II).

[27] For what it's worth, there are, in my view, two fallacies in Leibniz's argument. First, though it is true that Caesar is who he is in virtue of the properties he has, it is *not* true that Caesar is who he is in virtue of *all* the properties he has. Surely only *some* of the properties one has are individuative of that person. (Kripke argues – correctly, in my view – that only facts about one's conditions of origination are individuative of one.) Second, a fact about x may be *individuative* of x without being *definitive* of the *concept* of x. This is really another way of stating Kripke's point that conceivability and possibility don't coincide. The *substance* water is individuated by its chemical structure. But the *concept* of water is not *definable* in chemical terms.

A similar problem faces Kant's analysis of necessity. Kant says that *bachelors are unmarried* is necessary because the concept on the left side of the copula *contains* the concept on the right; and that, in general, necessity stems from *containment* relations holding among concepts.[28] So, like Leibniz and Hume, Kant has anticipated our analysis. But the problem with Kant's analysis is clear: the concept *water* doesn't contain the concept *hydrogen* (or *oxygen*), even though water itself contains hydrogen.

Also, as Quine pointed out[29], it must be said what is meant by "containment". The following proposition is necessarily true: *x's cardinality is greater than that of the rational numbers* is true only if *x is a class such that there is no bijection between its members and the members of a class C whose members can be put into a bijection with the whole numbers.* But in what sense does the concept *cardinality greater than the cardinality of the rational numbers* contain the concept *class such that there is no bijection between its members and the whole numbers*?

Kant actually said that propositions of mathematics, like the one just given, are *synthetic* because he thought that – apart from totally trivial propositions like *7=7* – the requisite containment relations among the concepts flanking the "=" are not to be found. But this shows the limitations of trying to understand necessity in terms of containment relations holding among *concepts.* It does not, *pace* Kant, show that mathematical truths are non-analytic.

There are two points of importance here. First, the containment-- or inclusion-analysis has a venerable history: some of the luminaries of our discipline have looked to it to explain necessity. Second, our analysis has some advantages over the analyses put forth by these luminaries. They sought to understand necessity in terms of an inclusion-relation among *concepts.* We know from Kripke that this is not the way to go. Thus we have analyzed necessity *not* as an inclusion-relation among concepts, but as an inclusion-relation among *properties.* Consequently, our analysis does not face the problems that bedevil the analyses of Leibniz, Hume, and Kant.

[28] Kant 1965/1787 (48-54).
[29] Quine1990/1951 (26).

ABSTRACT

I argue, on both logical and epistemological grounds, that the possible-worlds analysis of necessity is false. And I provide an alternative analysis: necessarily true propositions are those that hold entirely in virtue of facts about the decomposition of properties. So *anything that weighs more than ten lbs weighs more than nine lbs* is necessarily true because the property of weighing more than ten lbs decomposes into (inter alia) the property of weighing more than nine lbs. I consider cases of necessary truths that seem not to conform to my analysis (e.g. *if P, then if P or Q*) and argue that, properly analyzed, they do conform to it.

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